Extra Credit Problem for Homework #5:

A metal bell with natural resonance frequency $\omega_0$ is stuck once with a hammer for time $\Delta t$. We know from experience that the bell will in fact ring so that some resonant driving force is present when the bell is struck. We can get the frequency spectrum of the impulse by using the Fourier Transform:

$$A(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt$$

The quantity $A(\omega) d\omega$ gives the amplitude of the driving term with a frequency between $\omega$ and $\omega + d\omega$. Assuming the strength of the force is a constant $f_0$ acting over $\Delta t$, find $A(\omega)$ and sketch it. Derive an expression that relates the width of the relevant frequencies, $\Delta \omega$, to the pulse width $\Delta t$. If $\Delta t \to \infty$, meaning the force is constant, will the bell ring? Interpret this result. Now, as the force acts over a very short time $\Delta t \to 0$, will the bell ring? If a bell has frequency $\nu_0 = 1$ kHz, what is the maximum required $\Delta t$ to make a bell ring?