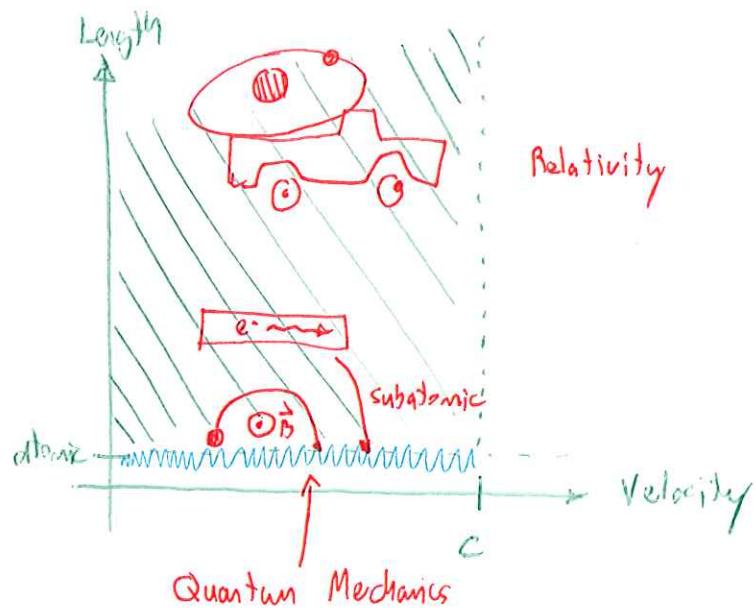


Physics 411 - Winter 2015

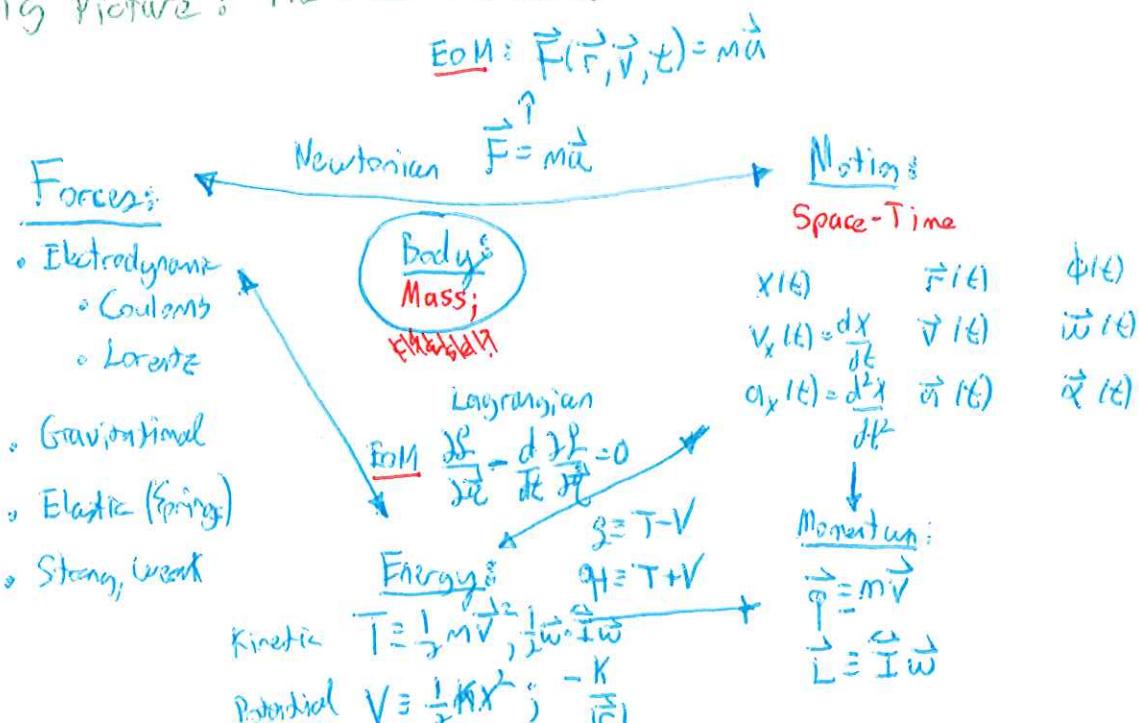
Monday, Week 1

- Welcome to Physics 411, which is a course in classical mechanics
 - Classical mechanics concerns itself with the motion of macroscopic bodies.
 - Macroscopic Bodies: The Domain of Mechanics (Show Powers of Ten)
 - Speed of an object (Macroscopic Body) is also relevant.



Classical Mechanics occupies a huge part of this parameter space!!!

- The Big Picture: The Main Characters



to discover & explore

The goal of this course is the stories that these characters tell about Nature,
of ~~creatures~~ that live in Nature & Nature itself.
creatures

We'll explore these stories by developing the language & tools necessary to navigate between these characters.

A main theme of this course is to arrive at the Equation of Motion (EoM) for a particular system & solve it:

$$\vec{F}(\vec{r}, \vec{v}, t) = m \cdot \vec{a}$$

$$\frac{\partial \vec{r}}{\partial t} = \vec{v}$$

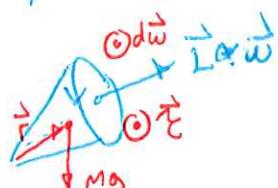
$$\frac{d\vec{L}(\vec{r}, \vec{v}, t)}{dt} - \frac{d}{dt} \frac{\partial L(\vec{r}, \vec{v}, t)}{\partial \vec{v}} = 0$$

This a course in Differential Equations as well, & also computational methods, as well as Physics.

- Small Groups :
- Name
 - What animal would you be & why?
 - Physics Question : Similarities between Earth & Spinning top
(PPT) (DEMO)

Precession of the Equinoxes

Why do they Precess?

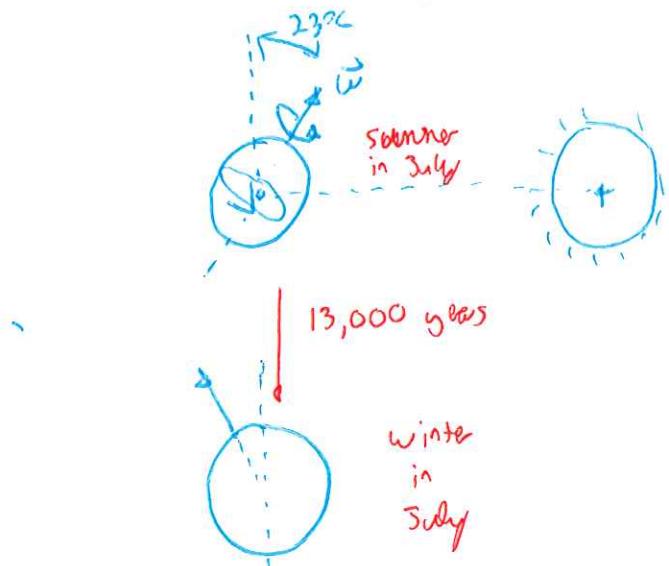


$$d\vec{\omega} = \frac{\vec{\tau}}{I} dt$$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

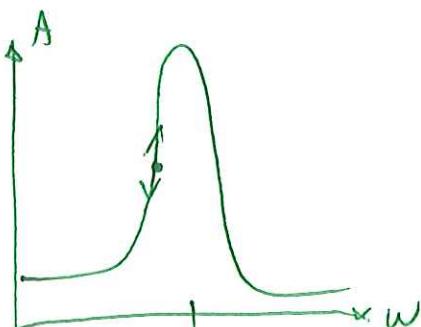
$$\vec{F} = \frac{\vec{v}}{t} = \vec{mV} = \vec{m}\vec{a} \rightarrow \vec{\tau} = \frac{\vec{v}}{L} = I\vec{\omega} = I \frac{d\vec{\omega}}{dt}$$

The Earth is also Precessing...



- Mechanics is relevant to modern research: My Research

- AFM



- Graphene NEVS: Detection, Mass sensing, degeneracy splitting

- CNT NEVS: Non-linearities

- Syllabus: Website & Forum (also written? in notecards)

- HW
 - Announcements
 - Additional reading

Physics 411 - Winter 2015

Wednesday, Week 1:

5 min Syllabus:

- Check website: aleman1@uoregon.edu/courses/physics-411-mechanics-winter-2015/
- Classical Mechanics by Taylor will be on website (first two chapters)
- MATH Handout (*Miriam + Kraige*)
- GTF Sabrina Moylechin.

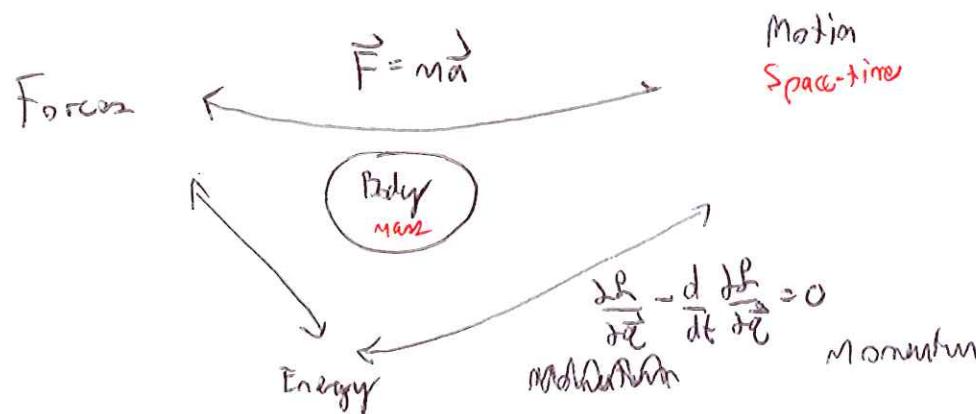
W 4-5] Wk. 72
 Th 2-3
 W 1-2 Drop-M

- HW Policy:
 - Neat
 - Late 1 day 25%
 - More ↑ 100%

- Course Project: Be creative, Examples, Analyze movie, note-cards, Extra chapter
- Forum or website for questions, & note cards last 3 minutes, office hours, class, etc.

5 min My Research

- My Goals:
 - Create safe & fun environment for you to explore physics,
 - Support you in achieving your dreams & goals
 - Be there for you as an instructor, mentor, & friend.



The three quantities that specify a physical system are:

- length (space) → Krypton-86 ($1,650,763,73 \text{ fm}$)
 \downarrow
 $\lambda = 2p^{10} \rightarrow 5d^5$ transition)

gives position
of particle or event



Now defined as length travelled by light in vacuum in $\frac{1}{c}$ seconds.
So defined by time standard.

- time → Cesium-133 hyperfine (spin) transition

\downarrow
dimension to order events
from past to future, & measure
of intervals between events.

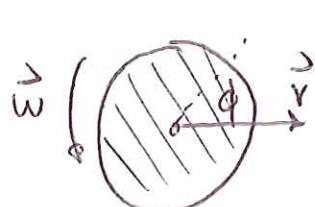
$\sim 10^{18}$ excitation; can generate w/ mechanical system
(9,26 Hz)

so time between 9.2×10^9 periods:

1 s

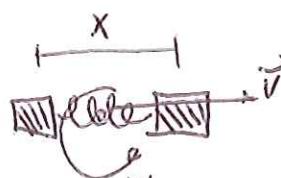
- mass → arb. standard: 1 kg is 1 l = 1000 cm^3 of water

First let's only treat point particles: Bodies → Point Particles:



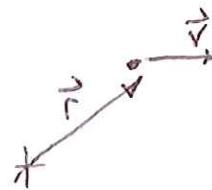
$\vec{\phi}, \vec{r}, t, \dots$

mass + extent



d, \vec{r}, x, t

mass + extent



\vec{r}, t

mass w/o extent
(particle)

- Assumptions (Axioms) concerning space + time:

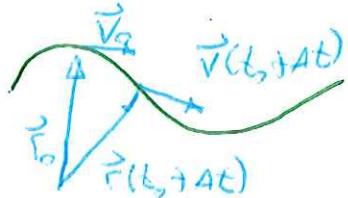
- Continuum
- Universal time scale (two observers of synchronized clocks are in agreement)
- Space is Euclidean (straight lines are shortest path between two points)
- Cartesian Coordinate Systems will be used for reference frames to measure positions of bodies.

We will deal with reference frames that are mechanically equivalent, meaning the physical laws responsible for motion are the same in both reference frames.

j? When are two reference frames mechanically equivalent? Is there some feature or characteristic that makes them so?

Consider that a particle's trajectory can be computed knowing \vec{r}_0, \vec{v}_0 & \vec{a} at each time t over an interval $\Delta t \rightarrow dt$:

\vec{a} contains information regarding the laws of motion



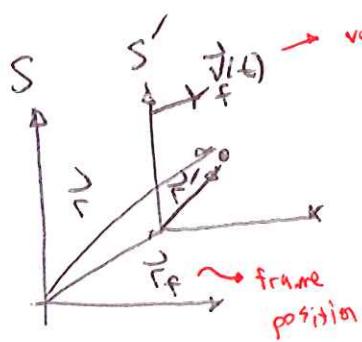
$$\vec{r}(t_0 + \Delta t) = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2} \left. \frac{d^2 \vec{r}}{dt^2} \right|_{t_0} \Delta t^2$$

$$\vec{v}(t_0 + \Delta t) = \vec{v}_0 + \left. \frac{d\vec{v}}{dt} \right|_{t_0} \Delta t$$

To second order (Δt^2), \vec{a} is the agent responsible for changing \vec{v} & for changing \vec{r} from straight line motion. We call these agent factors:

$$\vec{F} \sim \vec{a}$$

Now consider two reference frames that move relative to one another in an arbitrary way, a particle moving is being viewed by observers in each:



velocity of the frame is can be anything; can be wild!!

$$\vec{r} = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\vec{r}' = \vec{r}'_0 + \vec{v}'_0 \Delta t + \frac{1}{2} \vec{a}' \Delta t^2$$

$$\vec{r}_f = \vec{r}_{f,0} + \vec{v}_{f,0} \Delta t + \frac{1}{2} \vec{a}_f \Delta t^2$$

Note:

Since $\vec{V}_f(t)$ is arbitrary, the two observers may not observe the same motion & thus could infer completely different

Laws of Motion & Physics.

- Ex: watching stars "rotate" from earth \Rightarrow forces on

stars make them rotate!!



From, $\vec{r} = \vec{r}' + \vec{r}_f$ |
|

agree on location

$$\vec{v}'_o = \vec{v}_o - \vec{v}_{f,o}$$

$$\vec{r}_o + \vec{v}_o \Delta t + \frac{1}{2} \vec{\alpha} \Delta t^2 = \vec{r}'_o + \vec{v}'_o \Delta t + \frac{1}{2} \vec{\alpha}' \Delta t^2$$

$$+ \vec{r}_{f,o} + \vec{v}_{f,o} \Delta t + \frac{1}{2} \vec{\alpha}_f \Delta t^2$$

$$\vec{r}_o + \frac{1}{2} \vec{\alpha} \Delta t^2 = (\vec{r}'_o + \vec{r}_{f,o}) + \frac{1}{2} (\vec{\alpha}' + \vec{\alpha}_f) \Delta t^2$$

$$\text{so } \vec{r}_o = \vec{r}'_o + \vec{r}_{f,o} \quad (\text{trivial})$$

$$\vec{\alpha} = \vec{\alpha}' + \vec{\alpha}_f$$

Since we're letting $\vec{\alpha}_f$ be arbitrary, $\vec{\alpha}'$ dictates trajectory in S' frame, observers in S & S' will not conclude same physics unless $\vec{\alpha}_f = 0$.

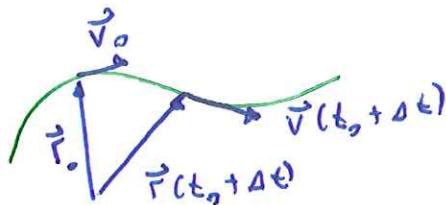
i.e. they will not be mechanically equivalent

(4)

Physics 411 - Winter 2015

Friday - Week 1 :

- Review:



straight line
parametrized by Δt

$$\vec{r}(t_0 + \Delta t) = \vec{r}_0 + \vec{v}_0 \Delta t + \frac{1}{2} \vec{a} \Delta t^2$$

$$\vec{v}(t_0 + \Delta t) = \vec{v}_0 + \vec{a} \Delta t$$

constant
vector

- \vec{a} causes deviation from straight trajectories if changes in velocity.
- Details about the Laws of Motion are contained completely in \vec{a} .
- Seems obvious, but only if you knew calculus (which Newton invented for his physics)
- The agents responsible for motion, which we call forces (\vec{F}), are proportional to \vec{a} :

$$\vec{F} \sim \vec{a}$$

(Newton would later use experimental data to find proportionality constant, mass)

- Two observers measuring different \vec{a} 's would infer different forces.

Inertial reference frames, where $\vec{a}_{\text{frame}} = 0$, are useful since observers in the frame will all agree on the same \vec{a} :

$$\vec{a} = \vec{a}' + \vec{a}_{\text{frame}}$$

$$\boxed{\vec{a} = \vec{a}'}$$

Thus, two frames are mechanically equivalent iff

$$\vec{a}_p = 0$$

This implies space will be homogeneous & isotropic, and that time will be homogeneous.

Physics (laws of motion) will not change at different positions in space or different directions in space.
 (time may not be isotropic... physics "looks" different when time is reversed)

Laws of Physics are rotationally & translationally invariant.

Ref. frames w/ $\vec{a}_p = 0$ are called inertial reference frames.

- * Later we'll see this invariance leads directly to the conservation of \vec{P} , \vec{L} , & Energy!!!

Vectors are a useful concept when ~~describing~~ ^{describing} the motion of dynamical systems
 position, velocity, forces, etc.

3D Vector Basis:

$$\vec{r} = (x, y, z) \quad \text{or} \quad \vec{r} = (\rho, \theta, r)$$

$$x, y, z \in \mathbb{R}$$

$$\phi \in [0, 2\pi]$$

$$\theta \in [0, \pi]$$

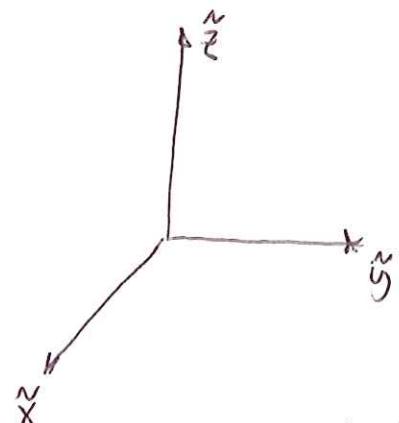
$$r \in [0, \infty)$$

Live in 3D right-handed
 Cartesian coordinate system

$$\vec{r} = x \hat{x} + y \hat{y} + z \hat{z}$$

$\rightarrow (r, \theta, \phi)$

$\downarrow (1, 0, 0) \quad (0, 1, 0)$



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$= \sum_{i=1}^3 r_i \hat{e}_i$$

$$= (r_x, r_y, r_z)$$

$$\text{Ex: } \vec{r}, \vec{v} = \frac{d\vec{r}}{dt}, \vec{a} = \frac{d^2\vec{r}}{dt^2}$$

\vec{F} (force), \vec{p} , \vec{L}

$$\vec{F}(\vec{r})$$



vector-valued function

Math w/ Vectors: $\vec{a}, \vec{b} \in \mathbb{R}^3$

$$\begin{aligned}\text{Addition: } \vec{a} + \vec{b} &= \sum_i (a_i + b_i) \hat{e}_i \\ &= (a_1 + b_1, \dots, a_3 + b_3) \\ &= \vec{c} \in \mathbb{R}^3 \quad [\text{Must be true for vector space}]\end{aligned}$$

$$\text{Subtraction: } \vec{a} - \vec{b} = \sum_i (a_i - b_i) \hat{e}_i$$

$$\text{Scalar Multiplication: } c \in \mathbb{R} \quad \vec{a} \in \mathbb{R}^3$$

$$c\vec{a} = \sum c a_i \hat{e}_i$$

$$= (ca_1, ca_2, ca_3)$$

$$\in \mathbb{R}^3 \quad [\text{Must be true for vector space}]$$

Vector Multiplication :

• Scalar or dot product

$$\mathbb{R}^3 \times \mathbb{R}^3 \rightarrow \mathbb{R}$$

$$\begin{aligned}\vec{a} \cdot \vec{b} &= a_1 b_1 + a_2 b_2 + a_3 b_3 \\ &= \sum_{i=1}^3 a_i b_i\end{aligned}$$

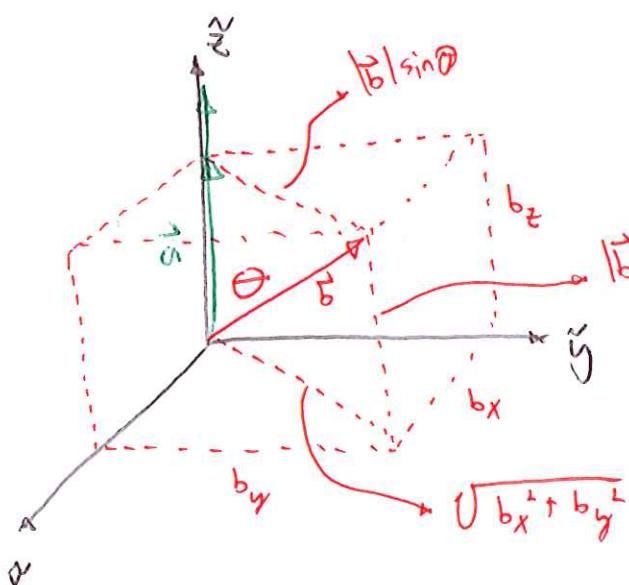
$$\begin{aligned}\vec{a} \cdot \vec{a} &= a_1^2 + a_2^2 + a_3^2 \\ &= |\vec{a}|^2\end{aligned}$$

$$\begin{aligned}\text{Ex: } T_B &= \frac{1}{2} \vec{w} \cdot \vec{L} \\ T_T &= \frac{1}{2} \vec{w} \cdot \vec{v} \cdot (\vec{m} \cdot \vec{v})\end{aligned}$$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad \begin{array}{l} \text{[Modulus or Magnitude]} \\ \text{or Length of vector] } \end{array}, \quad w = \vec{F} \cdot \vec{x}$$

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} \quad \text{[Unit Vector]}$$

Geometric Interpretation: Take \hat{a} along \vec{a} $\{$ consider $\vec{a} \cdot \vec{b}$



$$\vec{a} \cdot \vec{b} = a \cdot b_x + a \cdot b_y + a \cdot b_z$$

$$|\vec{b}| \cos \theta = b_z = |\vec{a}| b_z$$

$$\boxed{\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta}$$

Orthogonality: (1)

\vec{a}, \vec{b} are orthogonal if (4)

$$\vec{a} \cdot \vec{b} = 0 \Rightarrow \theta = \frac{\pi}{2}$$

Cross or Vector Product

$$\mathbb{R}^3 \otimes \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - a_z b_y) \hat{x} + (a_x b_z - a_z b_x) \hat{y} + (a_x b_y - a_y b_x) \hat{z}$$

Geometric Interpretation : Consider same figure ($\vec{a} \times \vec{b}$)

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & a \\ b_x & b_y & b_z \end{vmatrix} = -a b_y \hat{x} + a b_x \hat{y} = (-a b_y, a b_x, 0)$$

$$|\vec{a} \times \vec{b}| = \sqrt{a^2 b_y^2 + a^2 b_x^2}$$

$$= |\vec{a}| \sqrt{b_x^2 + b_y^2}$$

Notes:

$$\cdot \vec{a} \times \vec{b} \perp \text{to } \vec{a} \text{ & } \vec{b}$$

$$\cdot \vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

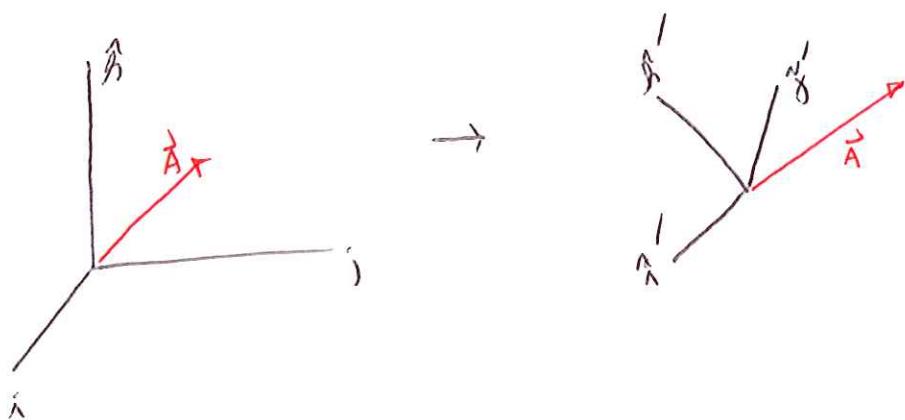
$$\text{Ex: } \vec{F} = \vec{r} \times \vec{f}$$

$$= \frac{d\vec{I}}{dt}$$

$$\vec{F} = q \vec{v} \times \vec{B}$$

$$\vec{I} = \vec{r} \times \vec{p}$$

Coordinate Transformations



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$= A'_x \hat{i}' + A'_y \hat{j}' + A'_z \hat{k}'$$

$$A'_x = \vec{A} \cdot \hat{i}'$$

$$= A_x \hat{i} \cdot \hat{i}' + A_y \hat{j} \cdot \hat{i}' + A_z \hat{k} \cdot \hat{i}'$$

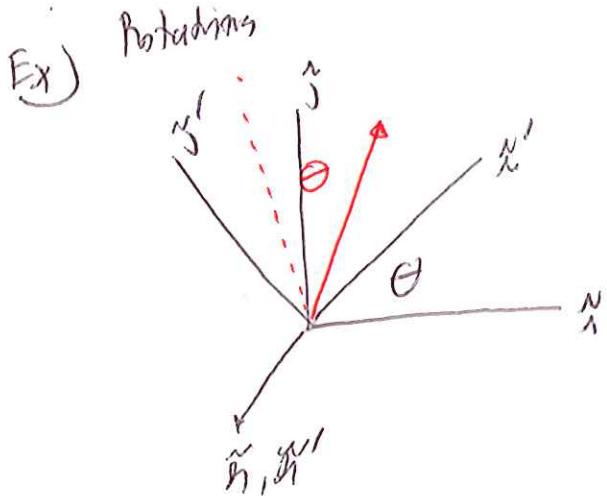
$$A'_y = A_x \hat{i} \cdot \hat{j}' + A_y \hat{j} \cdot \hat{j}' + A_z \hat{k} \cdot \hat{j}'$$

$$A'_z = A_x \hat{i} \cdot \hat{k}' + A_y \hat{j} \cdot \hat{k}' + A_z \hat{k} \cdot \hat{k}'$$

Transformation Matrix

$$\begin{pmatrix} A'_x \\ A'_y \\ A'_z \end{pmatrix} = \begin{pmatrix} \hat{i} \cdot \hat{i}' & \hat{j} \cdot \hat{i}' & \hat{k} \cdot \hat{i}' \\ \hat{i} \cdot \hat{j}' & \hat{j} \cdot \hat{j}' & \hat{k} \cdot \hat{j}' \\ \hat{i} \cdot \hat{k}' & \hat{j} \cdot \hat{k}' & \hat{k} \cdot \hat{k}' \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

T



$$T_R = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

For arbitrary ~~all~~ sequence of rotations:

$$T_R^N \cdots T_R^2 T_R^1$$

Vector Calculus:

Differentiation in one-dimension

$$\frac{df(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

Differentiation in Higher Dimensions

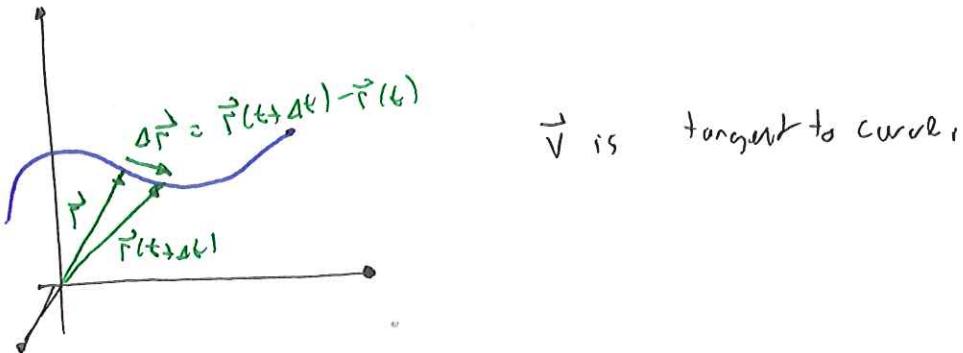
$$\mathbb{R} \rightarrow \mathbb{R}^3 \quad \mathbb{R}^m \rightarrow \mathbb{R}^3$$

$$\vec{r}(t) \quad \vec{f}(x, y, z, t)$$

$$\frac{d\vec{r}(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{\vec{r}(t + \Delta t) - \vec{r}(t)}{\Delta t}$$

$$= \vec{v}(t)$$

$$= \frac{d}{dt}$$



Properties:

$$\frac{d}{dt}(\vec{r} + \vec{s}) = \frac{d\vec{r}}{dt} + \frac{d\vec{s}}{dt} \quad [\text{Distributive}]$$

$$\frac{d}{dt}(f\vec{r}) = \frac{df}{dt}\vec{r} + f\frac{d\vec{r}}{dt} \quad [\text{Product Rule}]$$

$$\frac{d}{dt}(\vec{r} \cdot \vec{s}) = \vec{r} \cdot \dot{\vec{s}} + \vec{s} \cdot \dot{\vec{r}} \quad //$$

$$\frac{d}{dt}(\vec{r} \times \vec{s}) = \vec{r} \times \dot{\vec{s}} + \vec{s} \times \dot{\vec{r}} \quad //$$

$$\frac{d}{dt}(\vec{r}(u(s))) = u(s)\frac{d\vec{r}}{du} \quad [\text{Chain Rule}]$$

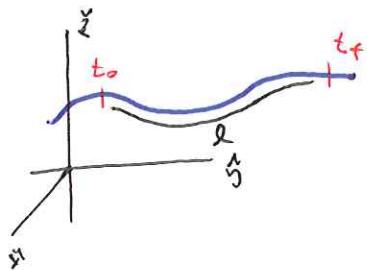
$$\begin{aligned} \frac{d}{dt}(\vec{r}(x, y, t)) &= \vec{x} \frac{d\vec{r}}{dx} + \vec{y} \frac{d\vec{r}}{dy} + \vec{z} \frac{d\vec{r}}{dt} \\ &= \vec{v} \cdot \vec{\nabla} \vec{r} + \frac{\partial \vec{r}}{\partial t} \end{aligned}$$

(8)

Differential Displacement vectors :

$$d\vec{r} = \left(\frac{d\vec{r}}{dt} \right) dt = \vec{v} dt$$

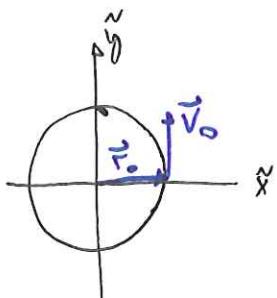
$$ds = |d\vec{r}| = |\vec{v}| dt$$



$$l = \int_{t_0}^{t_f} |\vec{v}| dt$$

Arc length
or
path length
of trajectory

Ex]



$$\vec{r} = \begin{pmatrix} r \cos t \\ r \sin t \end{pmatrix} \quad t \in [0, 2\pi]$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$= \begin{pmatrix} -r \sin t \\ r \cos t \end{pmatrix}$$

Path not a straight line \Rightarrow acceleration \Rightarrow force acting.

$$|\vec{v}| = \sqrt{r^2 \sin^2 t + r^2 \cos^2 t} = r \quad l = \int_0^{2\pi} r dt = 2\pi r$$

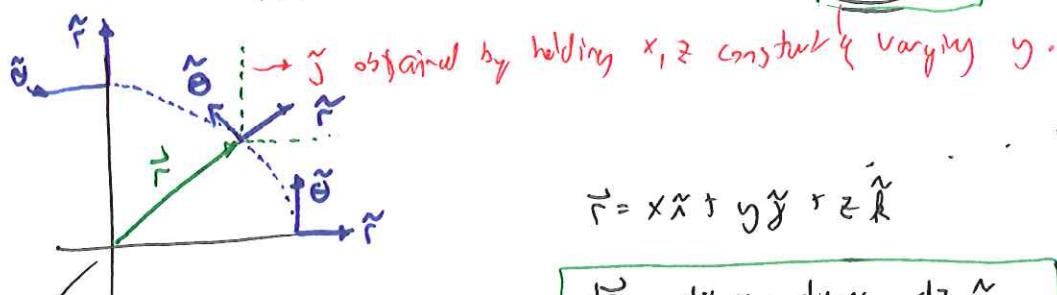
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Derivatives in other coordinate systems:

Later we will need to express derivatives of \vec{r} in other coordinate systems:

- Unit vectors $\hat{i}, \hat{j}, \hat{k}$ do not change in time:

$$\frac{d\hat{x}}{dt} = 0$$



$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{x} + \frac{dy}{dt}\hat{y} + \frac{dz}{dt}\hat{z}$$

(g)

- This is not true for polar coordinates for example; where $\hat{\theta}, \hat{r}$ clearly depend on t if $\vec{r}(t)$:

$$\begin{aligned}\vec{r} &= r_{\theta} \hat{\theta} + r_r \hat{r} \\ &= \begin{pmatrix} r_{\theta} \\ r_r \end{pmatrix}\end{aligned}$$

$$\frac{d\vec{r}}{dt} \neq \frac{dr_{\theta}}{dt} \hat{\theta} + \frac{dr_r}{dt} \hat{r} \quad \text{since } \dot{\theta} \neq 0 \quad \dot{r} \neq 0$$

We'll work this out later, but know $\hat{x}, \hat{y}, \hat{z}$ are special in this way.

Newton's Laws (1642-1726)

• History of Newton's Laws PPT

- Newton had calculus:

$$\vec{r}(at) = \vec{r}_0 + \vec{v}_0 at + \frac{1}{2} \vec{a} at^2 + \dots$$

$$\boxed{\begin{aligned}\vec{r}(t_0 + dt) &= \vec{r}_0 + \vec{v}_0 dt + \frac{1}{2} \vec{a} dt^2 \\ \vec{v}(t_0 + dt) &= \vec{v}_0 + \vec{a} dt\end{aligned}}$$

So Newton knew forces would be proportional to \vec{a} (He defined them so)



Mass is only property of Body left to us

Mass:

- characterizes an object's resistance to move
- $m > 0$