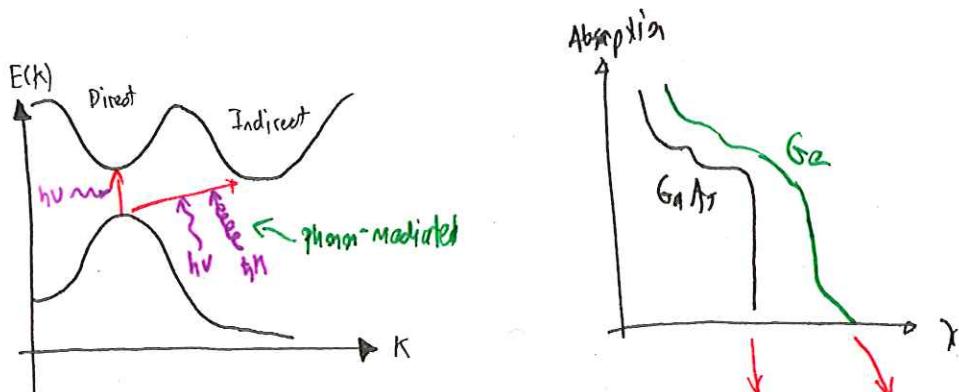
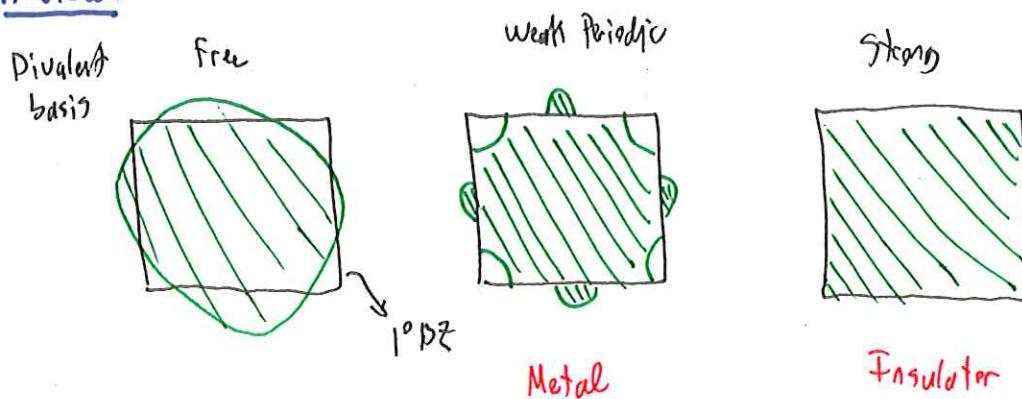


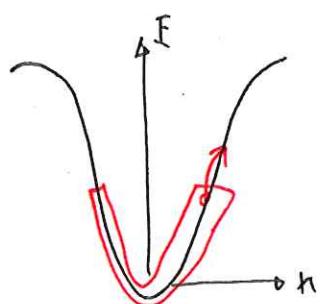
Physics 410/510 - Solid State Physics
 Spring 2015
Monday, Week 10

Review:

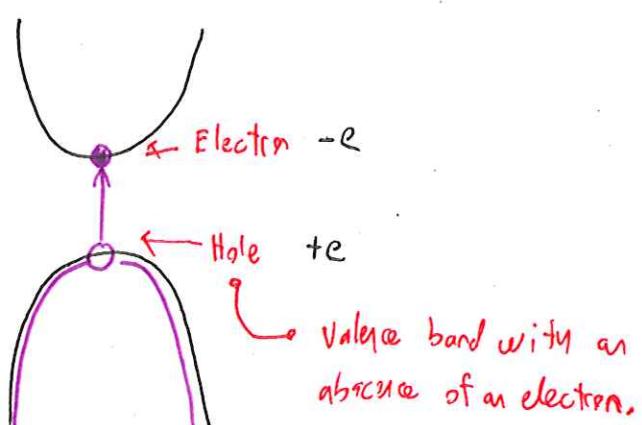


Bandgap energy gives the minimum absorption energy.

Intraband transitions in Metals give their color



Semiconductor Physics:

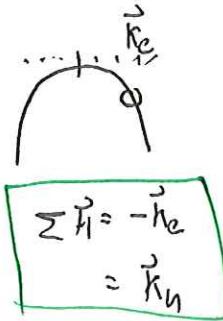


* Show PPT on Semiconductors

\vec{k}_{hole} :



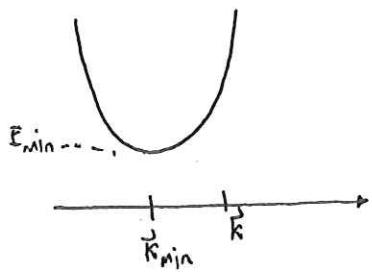
$$\sum \vec{k} = 0$$



$$\sum \vec{k} = -\vec{k}_h \\ = \vec{k}_h$$

Effective Mass:

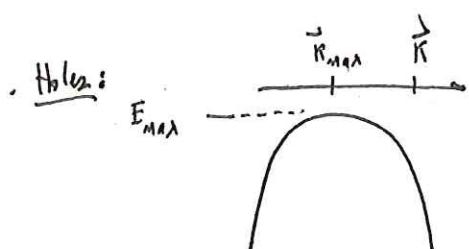
• Electrons:



$$E = E_{\min} + \kappa |\vec{k} - \vec{k}_{\min}|^2$$

$$\frac{\hbar^2}{m^*} = \frac{\partial^2 E}{2\hbar^2} = 2\kappa$$

$$m^* = \frac{\hbar^2}{\partial^2 E / \partial \vec{k}^2}$$

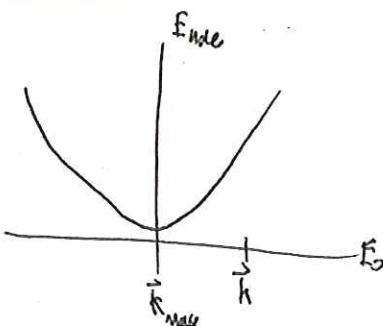


$$\text{for Electrons: } E_e = E_{\max} - \alpha |\vec{k} - \vec{k}_{\max}|^2$$

$m_{\text{hole}}^* > 0$ by definition, so

$$m_{\text{hole}}^* = -\frac{\hbar^2}{2E_e/\partial \vec{k}^2} > 0$$

$$E_{\text{hole}} = E_0 + \frac{\hbar^2 |\vec{k} - \vec{k}_{\max}|^2}{2m_{\text{hole}}^*}$$



EOM for electrons & holes:

$$m_e^* \frac{d\vec{v}}{dt} = -e(\vec{E} + \vec{v} \times \vec{B}) - m_e^* \frac{\vec{v}}{\tau}$$

$$m_h^* \frac{d\vec{v}}{dt} = +e(\vec{E} + \vec{v} \times \vec{B}) - m_h^* \frac{\vec{v}}{\tau}$$

In steady state, $|e\vec{E}| = m_e^* |\vec{v}|$ $|\vec{v}| = \frac{eE\tau}{m_e^*}$ [Drift Velocity]

we define

$$\boxed{\mu \equiv \frac{|\vec{v}|}{(E)} = \frac{e\tau}{m^*}} \quad [\text{Mobility}]$$

The mobility is a measure of how well the charge carrier moves in the crystal.

In some semiconductors, the dominant charge carriers are holes, which explains why

the Hall coefficient changed sign.

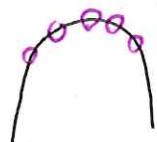
Impurity Doping:

Consider a pure band insulator: Without impurities we call it an intrinsic semiconductor



$$n \equiv \# e^-$$

$$n = p$$



$$p \equiv \# \text{ density holes}$$

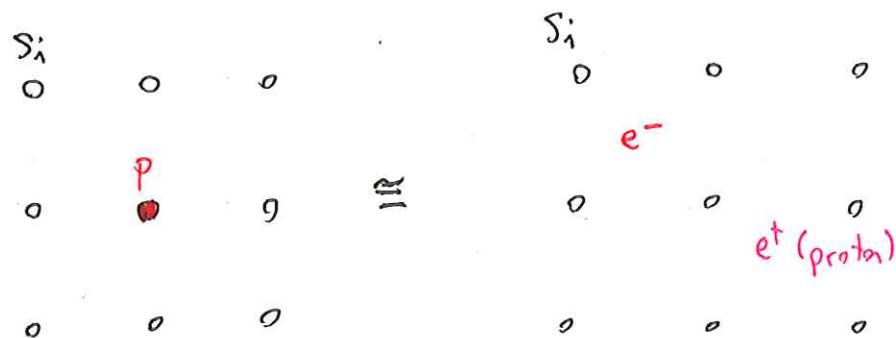


For intrinsic semiconductors

n-doping:

Now imagine adding an impurity from Group V elements to a Group IV element. The IV will have one more "free" electron than the IV,

Ex) Si $\stackrel{?}{\approx}$ P dopant



We can view P dopant as Si atom w/ extra electron plus a proton.



Not a hole; it doesn't live in the valence band.

The extra electron must go into the conduction band since the valence band is full.



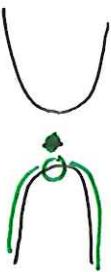
P atom \equiv n-dopant \Rightarrow density of e^- in conduction band.
 \equiv Donor (electron donor)

p-doping:

Add Group III, like Al, to Group IV, like Si:



In the case of Al, a hole is generated which lives in the valence band:



Al atom = p-dopant

= Acceptor

Impurity States: The electron & the positive core of an n-doped donor atom form a bound state.

$$V(r) = \frac{e^2}{4\pi\epsilon_r\epsilon_0} \cdot \frac{1}{r} \quad (\text{screening})$$

charge e^-
mass M_e^+

$$\begin{aligned} F_n^{eff} &= \frac{M_e^+ e^2}{8(\epsilon_r \epsilon_0)^2 h^2} \cdot \frac{1}{n^2} \\ &= R_y \left(\frac{M_e^+}{M} \frac{1}{\epsilon_r^2} \right) \cdot \frac{1}{n^2} \\ &= R_y \frac{1}{n^2} \end{aligned}$$

$R_y \approx 19.6 \text{ eV}$

relative permittivity
relative dielectric constant

$$\begin{aligned} a_0^{eff} &= \left(\frac{4\pi G_F h^2}{M e^2} \right) \cdot \left(\epsilon_r \frac{M}{M_e^+} \right) \\ &= a_0 \left(\epsilon_r \frac{M}{M_e^+} \right) \end{aligned}$$

0.5 \AA

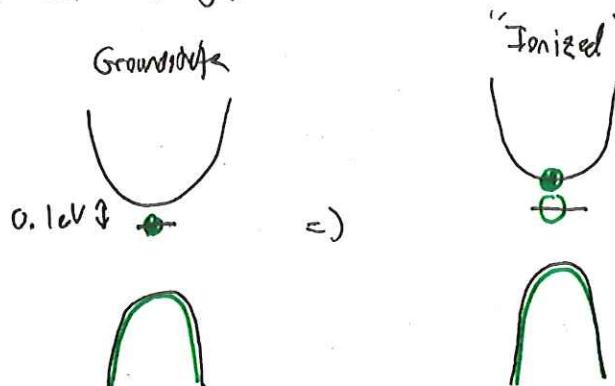
(5)

In Si:

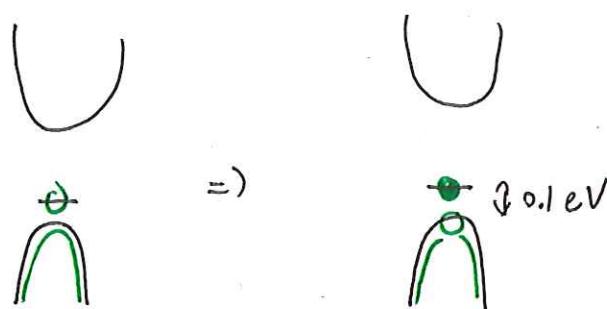
$$\beta_y^{\text{eff}} = 0.1 \text{ eV} < \frac{1}{100} \beta_y$$

$$\alpha_0^{\text{eff}} \approx 30 \text{ \AA} = 60 a_0$$

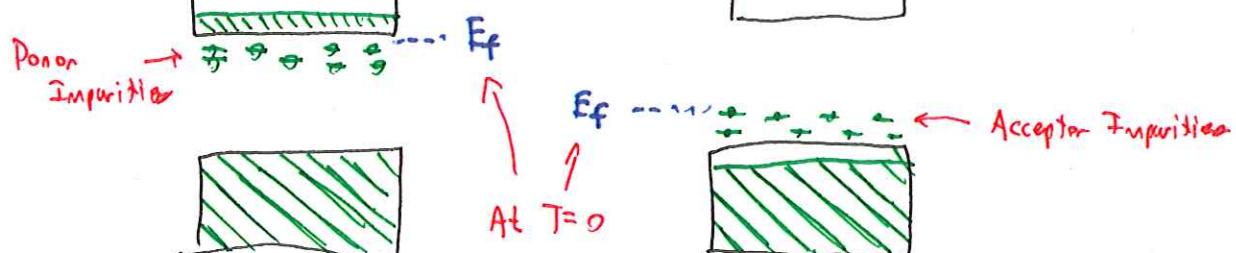
Since the electron will sit in the conduction band, the energy eigenstate of the donor will sit just below the conduction band:



For the case of an acceptor, we reverse the position of eigenstate so it sits near the top of the valence band



Energy Diagrams
of Doped Semiconductors
@ Room T



Physics 410/510 – Solid State Physics

Spring 2015

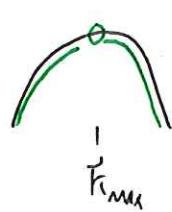
Wednesday, Week 10

Review:



$$E_e = E_{MAX} + \frac{\hbar^2}{2m_e^*} |\vec{k} - \vec{k}_{MAX}|^2$$

$$m_e^* = \frac{\hbar^2}{2E_g / 2\hbar^2}$$



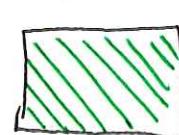
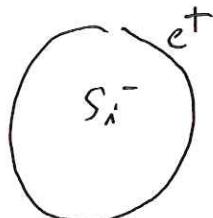
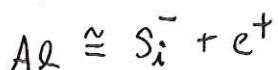
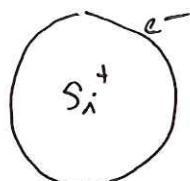
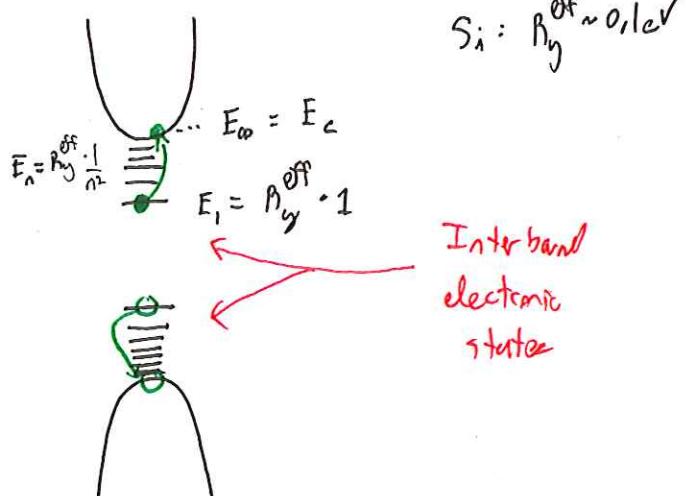
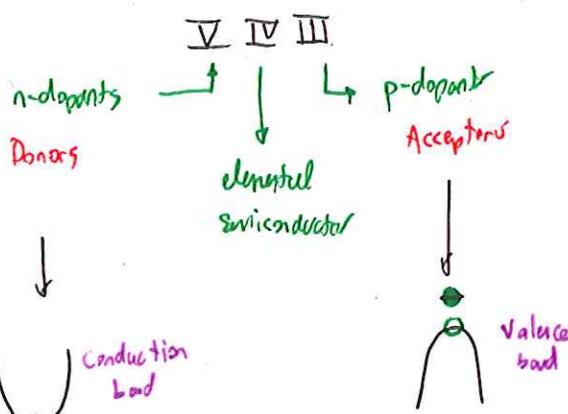
$$E_h = E_{MIN} + \frac{\hbar^2}{2m_h^*} |\vec{k} - \vec{k}_{MIN}|^2$$

$$\begin{aligned} m_h^* &= -\frac{\hbar^2}{2E_g / 2\hbar^2} > 0 \\ &= \frac{\hbar^2}{2E_u / 2\hbar^2} > 0 \end{aligned}$$

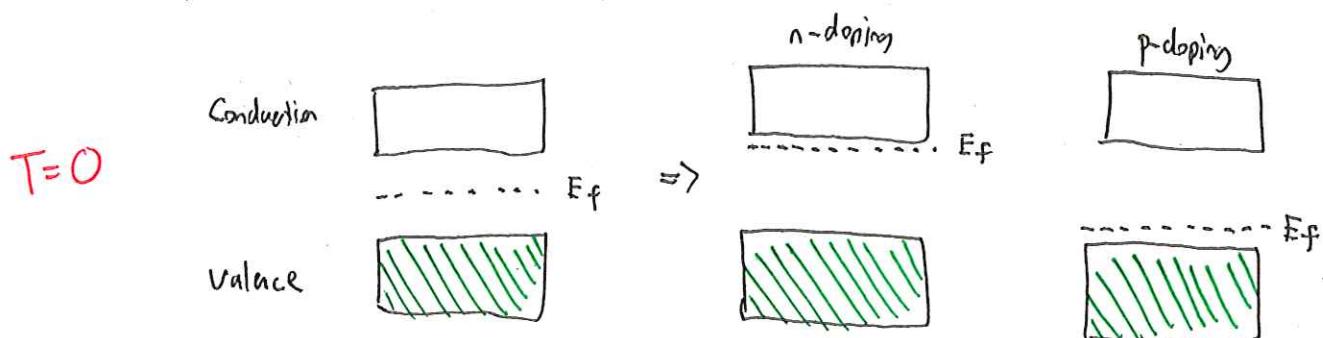
$$\mu = \frac{|V|}{|\vec{E}|} = \frac{e\tau}{m^*} \quad [\text{Mobility}]$$

Impurity Doping:

Group

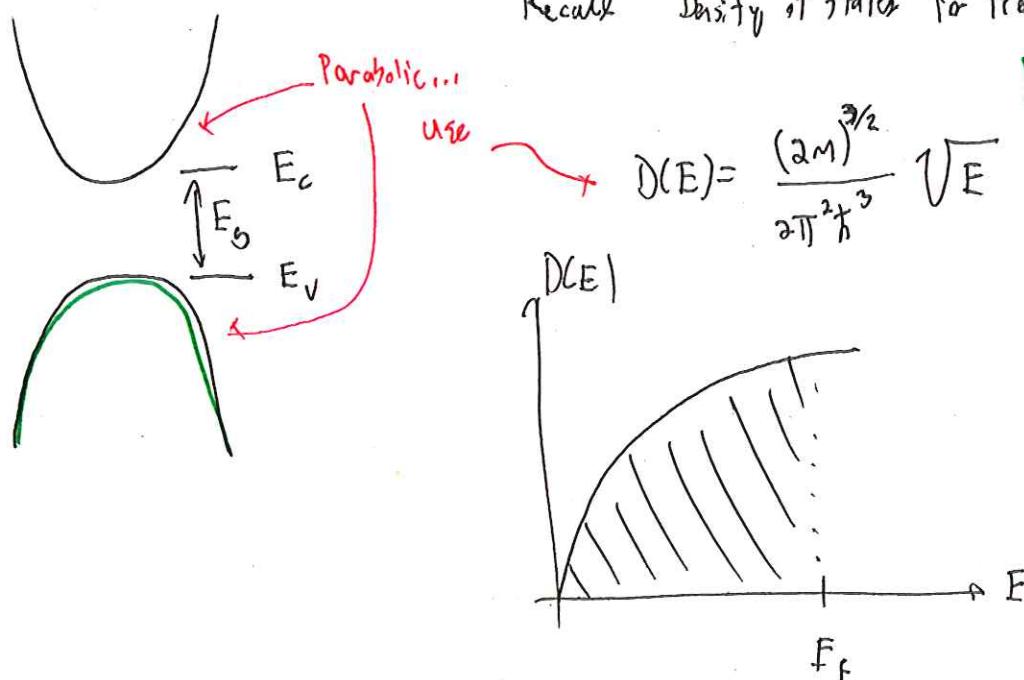


- At low T , the carriers ^{dopant} get bound to their respective nuclei in a process called "carrier freeze out".
- At $T=0$ for an intrinsic semiconductor, E_f sits half way between Conduction & valence band. Doping shifts E_f near the conduction (n -doping) or valence band (p -doping):



- Impurity states can dramatically change optical properties (i.e. optical absorption)

Statistical Mechanics of Semiconductors :



We can use this expression for electrons & holes:

e^-

$$D_c(E) = \frac{(2M_c^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_c}$$

$$E \geq E_c$$

e^+

$$D_v(E) = \frac{(2M_h^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E_v - E}$$

$$E \leq E_v$$

At fixed μ , n : (Intrinsic carrier concentration)

$$n(T) = \int_{E_c}^{\infty} D_c(E) f_{FD}(\beta(E-\mu)) dE = \int_{E_c}^{\infty} \frac{D_c(E)}{e^{\beta(E-\mu)} + 1} dE$$

$$\beta = \frac{1}{k_b T}$$

We assume $\mu < E_c \Rightarrow E > \mu \Rightarrow \beta(E-\mu) \gg 1 \Rightarrow$

This is the semiconductor/insulator assumption

$$e^{\beta(E-\mu)} + 1 \approx e^{-\beta(E-\mu)}$$

"Low T" limit; so thermal occupation is few $\approx E(T) \propto (k_b T)^{-1}$
i.e. parabolic

$$n(T) = \int_{E_c}^{\infty} e^{-\beta(E-\mu)} D_c(E) dE$$

$$= \frac{(2M_c^*)^{3/2}}{2\pi^2 \hbar^3} \int_{E_c}^{\infty} e^{-\beta(E-\mu)} (E - E_c)^{1/2} dE \times e^{-\beta E_c} \cdot e^{\beta E_c}$$

$$= \frac{(2M_c^*)^{3/2}}{2\pi^2 \hbar^3} e^{\beta(\mu - E_c)} \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-\beta(E-E_c)} dE$$

with $y^2 \equiv E - E_c$,

$$\begin{aligned}
 & \int_{E_c}^{\infty} (E - E_c)^{1/2} e^{-\beta(E-E_c)} dE \\
 &= 2 \int_0^{\infty} y e^{-\beta y^2} dy \\
 &= -2 \frac{d}{dp} \left(\int_0^{\infty} e^{-\beta y^2} dy \right) \\
 &= -\frac{d}{dp} \sqrt{\frac{\pi}{\beta}} \\
 &= \frac{1}{2} \sqrt{\pi} \beta^{-3/2}
 \end{aligned}$$

An unoccupied state in the valence band has a probability of being occupied by an electron given by f_{FD} . Thus the prob. of being unoccupied is

$$1 - f_{FD}$$

\Rightarrow

$$n(T) = \frac{1}{4} \left(\frac{2M_e^* k_B T}{\pi \hbar^2} \right)^{3/2} e^{-\beta(E_c - \mu)}$$

Density of electrons in the conduction band.

$$p(T) = \int_{-\infty}^{E_v} dE D_V(E) \left[1 - \frac{1}{e^{\beta(E-\mu)} + 1} \right] = \int_{-\infty}^{E_v} D_V(E) \frac{e^{\beta(E-\mu)}}{e^{\beta(E-\mu)} + 1} dE$$

If $\mu > E_v$, then $e^{\beta(E-\mu)} \ll 1$ so

$$p(T) = \int_{-\infty}^{E_v} D_V(E) e^{\beta(E-\mu)} dE$$

$$p(T) = \frac{1}{4} \left(\frac{2M_h^* k_B T}{\pi \hbar^2} \right)^{3/2} e^{-\beta(\mu - E_v)}$$

Density of holes in the valence band.

Law of Mass Action:

$$n(T)p(T) = \frac{1}{2} \left(\frac{k_B T}{\pi \hbar^2} \right)^{3/2} \left(M_e^* M_h^* \right)^{3/2} e^{-\beta(E_g - E_c - E_v)}$$

True independent of doping of material (p and/or n doping) !!

For an intrinsic semiconductor, $n = p$, so the $n(T)/p(T)$ gives:

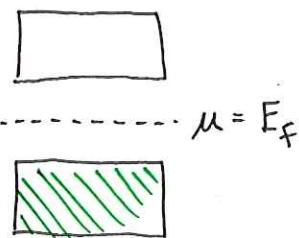
$$\log \left[\frac{n(T)}{p(T)} = 1 = \left(\frac{M_e^*}{M_h^*} \right)^{3/2} e^{-\beta(E_v + E_c - 2\mu)} \right]$$

$$= 0 \\ = \frac{3}{2} \log \left(\frac{M_e^*}{M_h^*} \right) - \beta(E_v + E_c - 2\mu)$$

$$\Rightarrow \boxed{\mu(T) = \frac{1}{2} (E_c + E_v) + \frac{3}{4} k_B T \log \left(\frac{M_h^*}{M_e^*} \right)}$$

Chemical Potential

? @ low T, $\boxed{\mu = \frac{1}{2} \frac{E_g}{E_v + E_c}}$



Also, the law of mass action gives:

$$n_{\text{intrinsic}} = p_{\text{intrinsic}}$$

$$= \sqrt{n_p p} \\ = \left[\frac{1}{V^2} \left(\frac{k_B T}{\pi \hbar^2} \right)^{3/2} \left(M_e^* M_h^* \right)^{3/4} e^{-\beta E_0/2} \right]$$

Intrinsic carrier concentration (electrons & holes)

Thus, the conductivity of electrons is:

$$\sigma_e = \frac{neV_e}{E} = n e \mu_e \times \text{mobility}$$

$$\propto e^{-E_0/2k_B T}$$

$$\left. \begin{aligned} \sigma &= \sigma_e + \sigma_h \\ \sigma_h &= \frac{peV_h}{E} = p e \mu_h \end{aligned} \right\}$$

$$\propto e^{-E_0/2k_B T}$$

Doped Semiconductors: Now let's add dopants (n & p , say) to the semiconductor & make the T high enough to activate the carriers. Then the doping, D will be:

$$D \equiv n - p$$

The number of charge carriers, in this regime, added by the dopants will remain constant, ^{essentially}

Physics 410/510 - Solid State Physics
Spring 2015
Friday, Week 10:

Review:



$$D_c(E) = \frac{(2M_e^*)^{3/2}}{2\pi^2 h^2} \sqrt{E_c - E}$$



$$D_v(E) = \frac{(2M_h^*)^{3/2}}{2\pi^2 h^2} \sqrt{E_v - E}$$

Density of states
per unit volume in
the conduction & valence
bands.

$$n(T) = \int_{E_c}^{\infty} D_c(E) f_{fp}(p(E-\mu)) dE = \boxed{\frac{1}{4} \left(\frac{2M_e^* k_B T}{\pi h^2} \right)^{3/2} e^{-\beta(E_c - \mu)}}$$

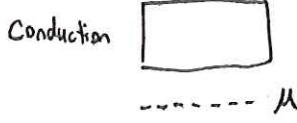
Carrier conc.
of el.

$$p(T) = \frac{1}{4} \left(\frac{2M_h^* k_B T}{\pi h^2} \right)^{3/2} e^{-\beta(\mu - E_v)}$$

carrier conc.
of holes.

$$n \cdot p = \frac{1}{2} \left(\frac{k_B T}{\pi h^2} \right)^3 (M_e^* M_h^*)^{3/2} e^{-\mu E_g}$$

Law of Mass Action



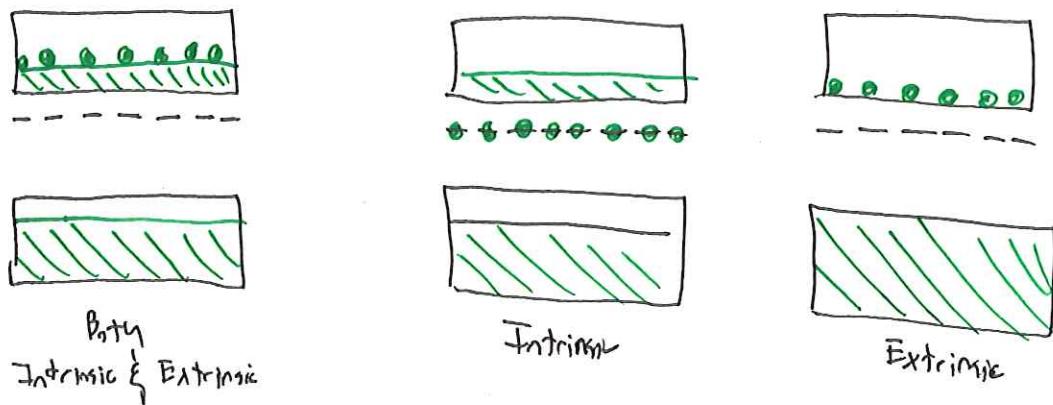
$$n_{\text{intrinsic}} = \frac{1}{\sqrt{2}} \left(\frac{k_B T}{\pi h^2} \right)^{3/2} (M_e^* M_h^*)^{3/4} e^{-E_g/2k_B T}$$

$$\sigma_h = \sigma_e \propto e^{-E_g/2k_B T} \Rightarrow \sigma \propto e^{-E_g/2k_B T}$$

Doped Semiconductors: Now add dopants ($n \neq p$ if you like) w/ T high enough to activate carriers.
The doping is

$$D = n - p$$

Carriers will be both intrinsic & extrinsic:



$$D = n - p$$

$$I = n_{\text{intrinsic}} = p_i$$

$$D^2 + 4I^2 = (n+p)^2$$

\Rightarrow

$$n = \frac{1}{2} \left(\sqrt{D^2 + 4I^2} + D \right)$$

$$p = \frac{1}{2} \left(\sqrt{D^2 + 4I^2} - D \right)$$

Semiconductor Device

Modern electronics constitute one of the greatest technological advances in the history of humankind, & owes its existence to the detailed understanding of quantum condensed matter physics.

Carrier concentrations
w/ intrinsic & extrinsic
carriers.

Band Structure Engineering:

- Designing Band Gaps

$$E_g^{\text{GaAs}} \approx 1.4 \text{ eV} \quad [\text{GaAs}]$$

$$E_g^{\text{AlAs}} \approx 2.7 \text{ eV} \quad [\text{AlAs}]$$

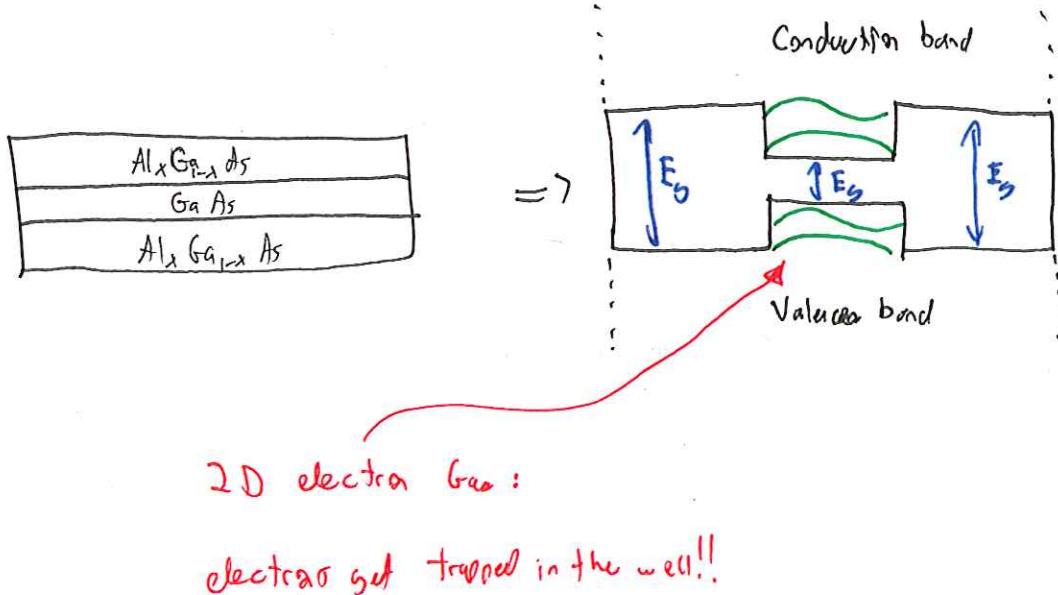
Alloys interpolate the band gap: $\underline{\text{Al}_x\text{Ga}_{1-x}\text{As}}$

$$E_g(x) = (1-x)1.4 + x \cdot 2.7 \quad [\text{eV}]$$

$$x < 0.4$$

useful for designing lasers!

- Semiconductor Heterostructure - The Quantum well

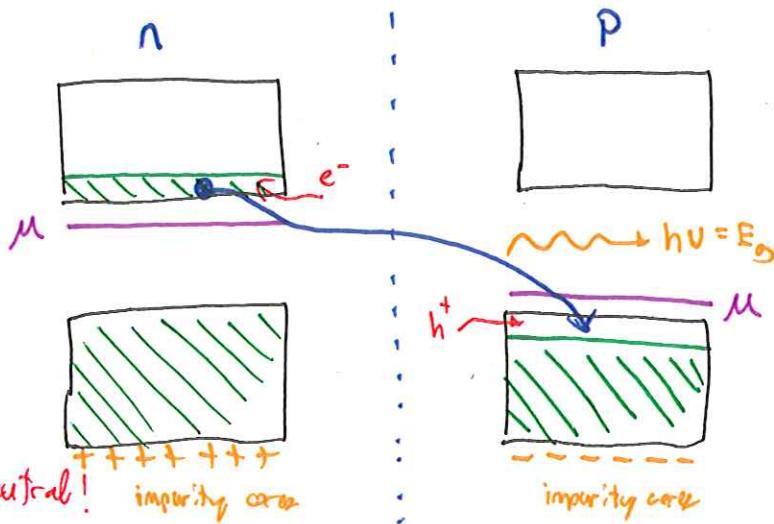


The p-n junction

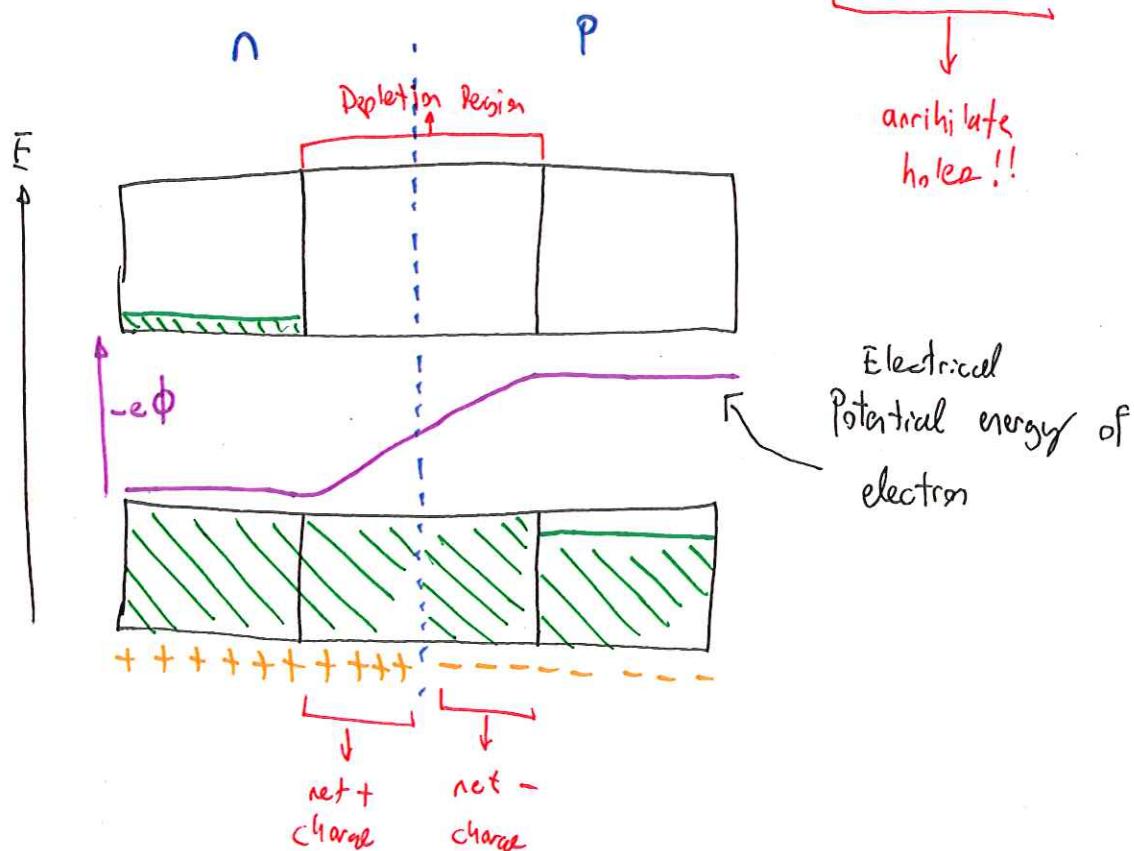
How do you do this?

Show PPT

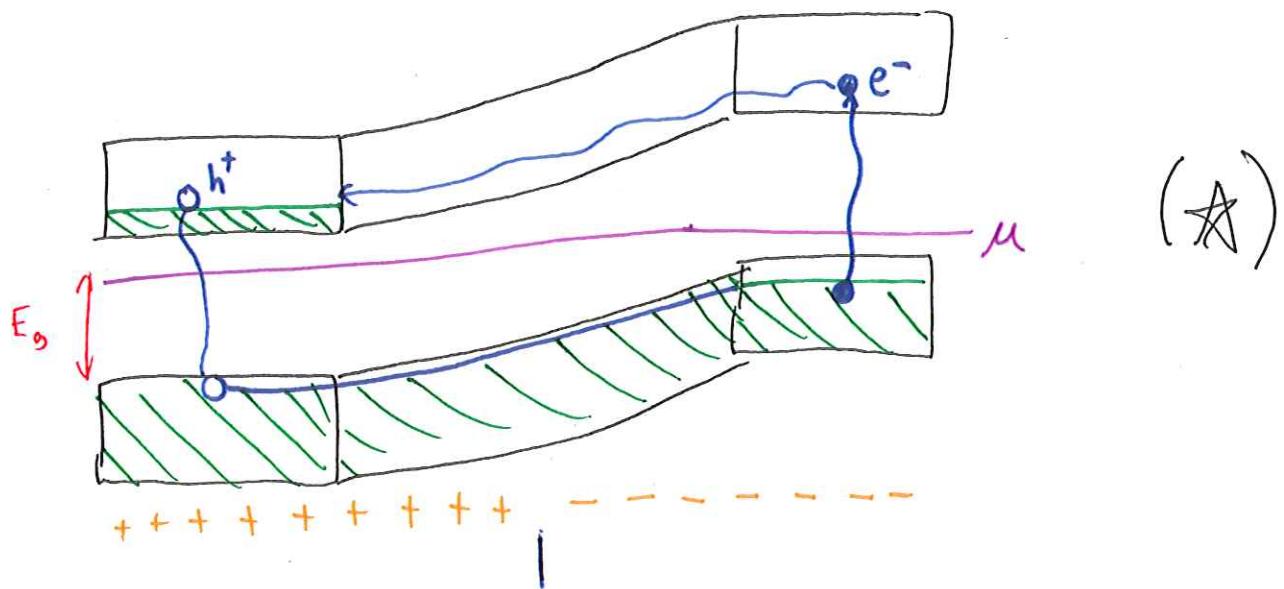
Contact a p-doped & n-doped semiconductor. First consider them separately:



Once in contact, electrons will migrate from n \rightarrow p sides, & h⁺ p \rightarrow n.



Including the potential ^{electrical} in the band diagram :

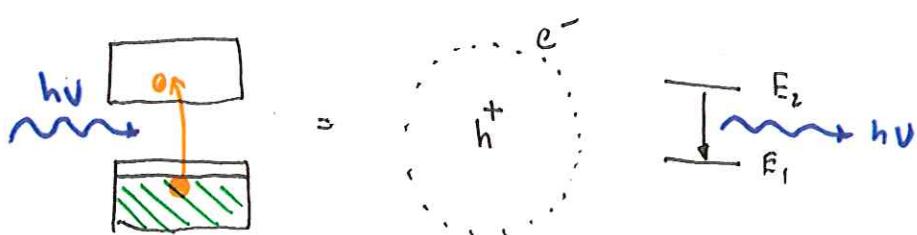


- In equilibrium, there will be small current due to ^{thermal} excitation of e^- & h^+ , but many of the $e^- - h^+$ pairs (excitons) will annihilate before moving.
- The chemical potential is now constant across the specimen, so no driving force exists.
 - The drop in the bandgap is now converted by change in electrostatic potential

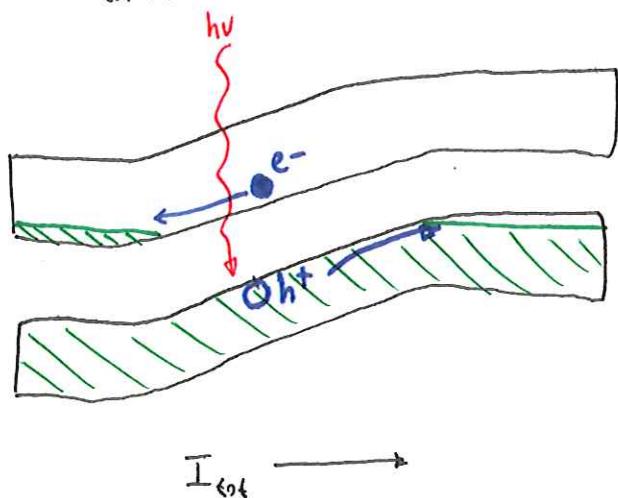


The Solar Cell:

In the picture (★), if $h\nu_{inc}$ is incident out of the depletion layer, excitons will mostly annihilate



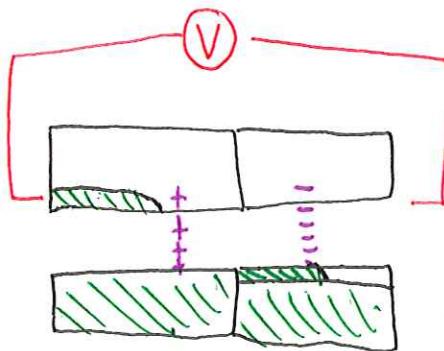
Illumination of the depletion region, however, will create exciton & the constituent e^- & h^+ will be transported away from the depletion region & generate current.



$$= -e \cdot n \cdot (-v) + e \cdot n \cdot (+v)$$

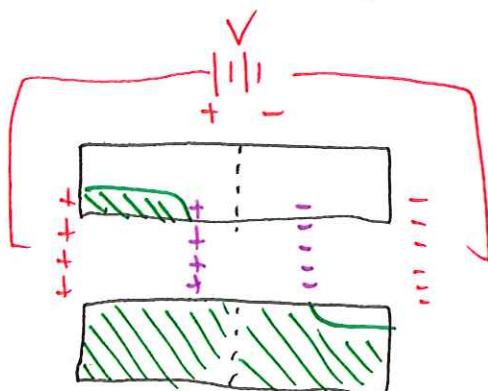
This is how photovoltaics & photo-detectors work!!

The Diode:



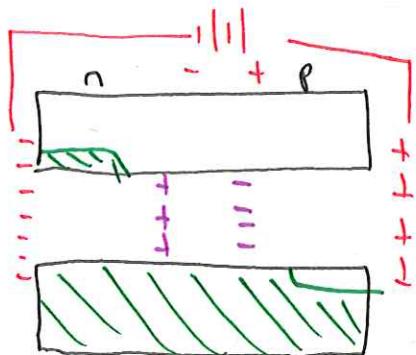
Depletion Region

Reverse Biased



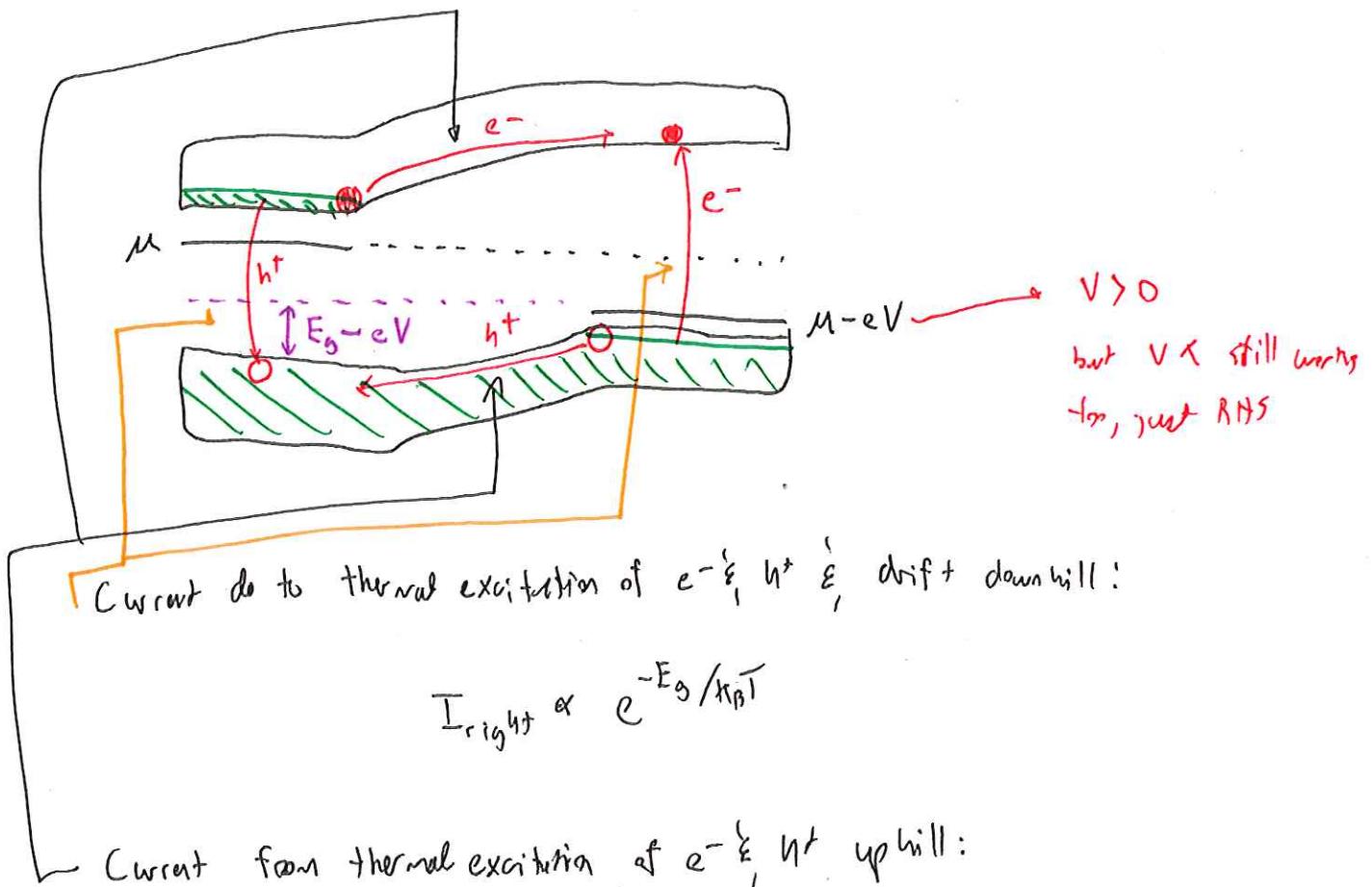
little current,
or No

Forward biased:



Current!!

The chemical potential is now no longer uniform across the device.



Constants of

proportionality

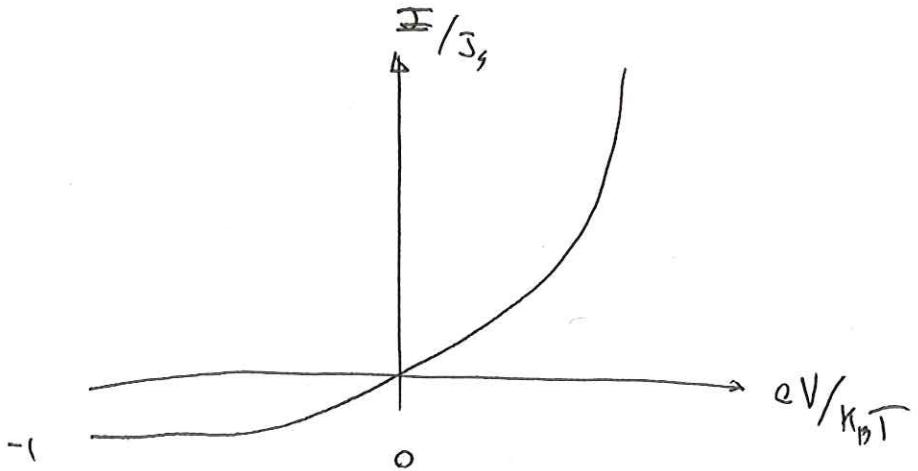
same for I_L & I_P

to make $I_{tot} \approx 0 @ V=0$

$$I_{left} \propto e^{-(E_g - eV)/k_B T} \quad \text{Saturation current}$$

$$I_{tot} = I_s(T) \left(e^{eV/k_B T} - 1 \right)$$

[Diode Eq.]



\Rightarrow Rectification : current flows only in one direction.

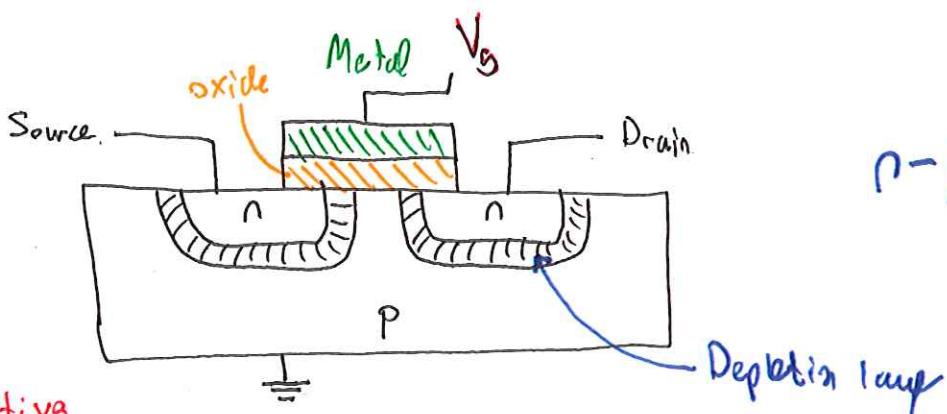
diode = "two party"

$$[n|p] = \rightarrow \text{diode symbol}$$

$$[p|n] = \left[\begin{array}{c} \rightarrow \\ \text{---} \end{array} \right] \text{diode symbol}$$

The Transistor : (foundation of modern electronics)

npn



n-MOSFET

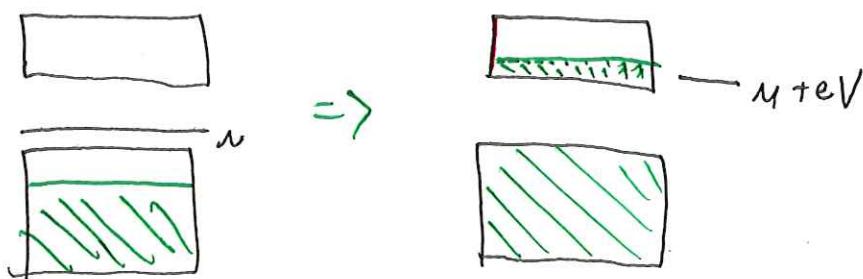
The effective circuit w/ $V_g = 0$?

$$\boxed{\text{---}} \quad || \quad \boxed{\text{---}}$$

(Two back-to-back diodes)

Saw with $V_{gate} = V_g = 0$, current doesn't flow easily!!

$V_g > 0$:



The charge carriers in the conduction band scale exponentially w/ $E_c - (\mu + eV)$:

$$n(T) \propto e^{-\frac{(E_c - (\mu + eV))}{k_B T}}$$

so the p-doped region becomes field-dependent very quickly past some $V_{threshold}$.

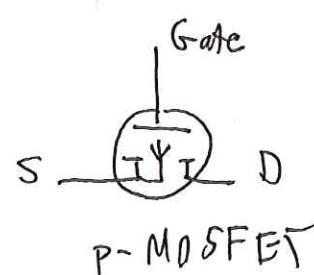
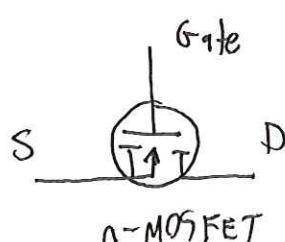
After

$$V_g > V_{threshold} :$$



{ current flows freely.

p-n-p Transistors are similar except for $V_{threshold} < 0$.



Together
=> CMOS
complementary.